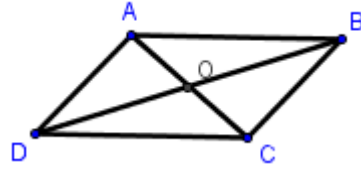


Exercice 1:

$$1. a) \text{ On a } AC^2 + BD^2 = \overline{AC}^2 + \overline{BD}^2$$

$$= (\overline{AB} + \overline{AD})^2 + (\overline{AD} - \overline{AB})^2$$

$$= \overline{AB}^2 + 2\overline{AB} \cdot \overline{AD} + \overline{AD}^2 + \overline{AD}^2 - 2\overline{AD} \cdot \overline{AB} + \overline{AB}^2$$

$$= AB^2 + 0 + AD^2 + AD^2 - 0 + AB^2$$

$$= 2(AB^2 + AD^2)$$

$$b) \text{ On a } AC^2 + BD^2 = 2(AB^2 + AD^2)$$

$$\text{donc } AB^2 = \frac{AC^2 + BD^2}{2} - AD^2 = \frac{6^2 + 4^2}{2} - \sqrt{7}^2 = 19 \text{ d'où } AB = \sqrt{19}$$

$$2. a) \text{ D'après le théorème d'ALKASHI}$$

$$\text{on a } \cos(\text{AOB}) = \frac{OA^2 + OB^2 - AB^2}{2 \times OA \times OB} = \frac{3^2 + 2^2 - \sqrt{19}^2}{2 \times 3 \times 2} = \frac{9 + 4 - 19}{12} = \frac{1}{2}$$

$$b) \text{ On a } \cos(\text{AOB}) = \frac{1}{2} \text{ donc } \text{AOB} = \frac{\pi}{3}$$

Exercice 2:

$$1. \text{ On a } (\overline{BA} + \overline{IB}) \cdot (\overline{BA} - \overline{IB}) = \overline{BA}^2 - \overline{IB}^2$$

$$= BA^2 - IB^2$$

$$= 4^2 - 4^2$$

$$= 0$$

$$2. \text{ On a } \overline{IA} \cdot \overline{ID} = (\overline{IB} + \overline{BA}) \cdot (\overline{IC} + \overline{CD}) \quad (\text{car } \overline{CD} = \overline{BA})$$

$$= (\overline{IB} + \overline{BA}) \cdot (-\overline{IB} + \overline{BA}) \quad \text{et } \overline{IC} = -\overline{IB})$$

$$= 0$$

$$3. \text{ D'après l'expression trigonométrique du produit scalaire}$$

$$\text{On a } \cos(\text{AID}) = \frac{\overline{IA} \cdot \overline{ID}}{IA \times ID} = \frac{0}{IA \times ID} = 0$$

$$\text{puisque } \cos(\text{AID}) = 0 \text{ donc } \text{AID} = \frac{\pi}{2}$$

