

$$F = \frac{4}{3} \times \frac{12}{15} - \frac{16}{3} \times \frac{6}{8} \text{ و } E = \frac{-2}{3} + \frac{7}{6} - \frac{1}{4} - 2 \text{ و } D = \frac{5 + \frac{1}{3}}{2 - \frac{3}{2}}$$

$$M = \left(\frac{2}{3} - \frac{5}{2}\right)^2 \quad H = \left(1 - \frac{5}{2}\right)^2 \quad G = \left(1 - \frac{1}{3}\right)\left(\frac{2}{5} + 1 - \frac{1}{2}\right)$$

$$N = [(a-c) - (a-b)] - [(c-a) + (b-c)]$$

الأجوبة:

$$A = \frac{3}{4} + \frac{5}{3} - \frac{7}{6} = \frac{9}{12} + \frac{20}{12} - \frac{14}{12} = \frac{9+20-14}{12} = \frac{15}{12} = \frac{5}{4}$$

$$B = 3(-12) - 5 + 14 - 10$$

$$B = -36 - 5 + 14 - 10 = -41 + 4 = -37$$

$$C = 3a - 5b + 5c - 2a - 4b - 3c - a - 2c = -9b$$

$$D = \frac{5 + \frac{1}{3}}{2 - \frac{3}{2}} = \frac{\frac{16}{3}}{\frac{1}{2}} = \frac{16}{3} \times \frac{2}{1} = \frac{32}{3}$$

$$E = \frac{-2}{3} + \frac{7}{6} - \frac{1}{4} - 2 = \frac{-8}{12} + \frac{14}{12} - \frac{3}{12} - \frac{24}{12} = \frac{-8+14-3-24}{12} = \frac{-21}{12} = -\frac{7}{4}$$

$$F = \frac{4}{3} \times \frac{12}{15} - \frac{16}{3} \times \frac{6}{8} = \frac{16}{15} - 4 = \frac{16-60}{15} = -\frac{44}{15}$$

$$G = \left(1 - \frac{1}{3}\right)\left(\frac{2}{5} + 1 - \frac{1}{2}\right) = \left(\frac{3}{3} - \frac{1}{3}\right)\left(\frac{4}{10} + \frac{10}{10} - \frac{5}{10}\right) = \frac{2}{3}\left(\frac{4+10-5}{10}\right) = \frac{2}{3}\left(\frac{9}{10}\right) = \frac{2 \times 3 \times 3}{3 \times 5 \times 2} = \frac{2}{5}$$

$$G = \frac{1}{2} \times \frac{1}{6} = \frac{1}{12}$$

$$H = \left(1 - \frac{5}{2}\right)^2 = \left(\frac{2-5}{2}\right)^2 = \left(\frac{-3}{2}\right)^2 = \frac{9}{4}$$

$$M = \left(\frac{2}{3} - \frac{5}{2}\right)^2 = \left(\frac{4-15}{6}\right)^2 = \left(\frac{-11}{6}\right)^2 = \frac{(-11)^2}{6^2} = \frac{121}{36}$$

$$N = [(a-c) - (a-b)] - [(c-a) + (b-c)] = (a-c-a+b) - (c-a+b-c)$$

$$N = a - c - a + b - c + a - b + c = a - c$$

تمرين 4: أحسب:

$$A = (200520052006)^2 - (200520052005 \times 200520052007)$$

الجواب: نلاحظ أن الأعداد الثلاثة تختلف فقط في رقم وحداتها

لتبسيط الحساب نضع: $x = 200520052006$

إذن: $200520052005 = x - 1$ و $200520052007 = x + 1$

$$A = x^2 - (x-1)(x+1)$$

$$= x^2 - (x^2 - 1)$$

$$= x^2 - x^2 + 1$$

$$= 1$$

إذن:

$$A = (200520052006)^2 - (200520052005 \times 200520052007) = 1$$

تمرين 1: باستعمال الرموز: \notin ; \in ; \subseteq ; \subset أملأ الفراغات التالية:

$$-7 \dots \mathbb{N} \text{ و } \frac{2}{3} \dots \mathbb{N} \text{ و } \sqrt{2} \dots \mathbb{N} \text{ و } \frac{8}{2} \dots \mathbb{N} \text{ و } \frac{15}{3} \dots \mathbb{N}$$

$$12 - 32 \dots \mathbb{N} \text{ و } \sqrt{25} \dots \mathbb{N} \text{ و } \frac{\sqrt{100}}{5} \dots \mathbb{N} \text{ و } 2, 12 \dots \mathbb{N}$$

$$\pi \dots \mathbb{N}^* \text{ و } 0 \dots \mathbb{N}^* \text{ و } \{1; 2; 7\} \dots \mathbb{N} \text{ و } \{4; -2; 12\} \dots \mathbb{N} \text{ و } \mathbb{N}^* \dots \mathbb{N}$$

الأجوبة:

$$12 - 32 \notin \mathbb{N} \quad -\frac{15}{3} \notin \mathbb{N} \quad \frac{8}{2} \in \mathbb{N} \quad \sqrt{2} \notin \mathbb{N} \quad \frac{2}{3} \notin \mathbb{N} \quad -7 \notin \mathbb{N}$$

$$\pi \notin \mathbb{N} \quad 2, 12 \notin \mathbb{N} \quad \frac{\sqrt{100}}{5} \in \mathbb{N} \quad \sqrt{25} \in \mathbb{N}$$

$$\mathbb{N}^* \subset \mathbb{N} \quad \{4; -2; 12\} \not\subset \mathbb{N} \quad \{1; 2; 7\} \subset \mathbb{N} \quad 0 \notin \mathbb{N}^*$$

تمرين 2: أحسب و بسط:

$$B = -3(-11) + 7 - 5 + 8 - 10 \text{ و } A = -\frac{5}{9} + \frac{5}{3} + \frac{7}{3}$$

$$C = 3a - 4b + c + 11a - 3b - 7c \text{ و}$$

$$F = \frac{4}{7} \times \frac{14}{6} - \frac{25}{8} \times \frac{3}{15} \quad E = \frac{3}{4} + \frac{5}{3} - \frac{7}{6} \text{ و } D = \frac{5-1}{3} - \frac{2}{7}$$

$$H = \left(1 - \frac{4}{3}\right)^2 \quad G = \left(1 - \frac{1}{2}\right)\left(\frac{2}{3} + 1 - \frac{3}{2}\right)$$

$$A = -\frac{5}{9} + \frac{5}{3} + \frac{7}{3} = -\frac{5}{9} + \frac{10}{3} = \frac{-5+30}{9} = \frac{25}{9}$$

$$B = -3(-11) + 7 - 5 + 8 - 10 = 33 + 7 - 5 + 8 - 10$$

$$B = 33 + 7 + 8 - 5 - 10 = 48 - 15 = 33$$

$$C = 3a - 4b + c + 11a - 3b - 7c = 14a - 7b - 6c$$

$$D = \frac{5-1}{3} - \frac{2}{7} = \frac{4}{3} - \frac{2}{7} = -\frac{2}{3} \times \frac{7}{6} = -\frac{7}{9}$$

$$E = \frac{3}{4} + \frac{5}{3} - \frac{7}{6} = \frac{9}{12} + \frac{20}{12} - \frac{14}{12} = \frac{9+20-14}{12} = \frac{15}{12} = \frac{5}{4}$$

$$F = \frac{4}{7} \times \frac{14}{6} - \frac{25}{8} \times \frac{3}{15} = \frac{4 \times 14}{7 \times 6} - \frac{25 \times 3}{8 \times 15} = \frac{4 \times 7 \times 5}{3 \times 3 \times 2} - \frac{4 \times 5 \times 32}{3 \times 8 \times 24} = \frac{17}{24} - \frac{17}{24}$$

$$G = \left(1 - \frac{1}{2}\right)\left(\frac{2}{3} + 1 - \frac{3}{2}\right) = \frac{1}{2}\left(\frac{2}{3} + \frac{3}{3} - \frac{3}{2}\right) = \frac{1}{2}\left(\frac{5}{3} - \frac{3}{2}\right) = \frac{1}{2}\left(\frac{10-9}{6}\right)$$

$$G = \frac{1}{2} \times \frac{1}{6} = \frac{1}{12}$$

$$H = \left(1 - \frac{4}{3}\right)^2 = \left(\frac{3-4}{3}\right)^2 = \left(\frac{-1}{3}\right)^2 = \frac{1}{9}$$

تمرين 3: أحسب و بسط:

$$B = 3(-12) - 5 + 14 - 10 \text{ و } A = \frac{3}{4} + \frac{5}{3} - \frac{7}{6}$$

$$C = 3a - 5b + 5c - 2a - 4b - 3c - a - 2c$$



التعميل هو كتابة
مجموع على شكل
جذاء

أجوبة:

$$A = 3x^2 - 3x = 3x(x - 1)$$

$$B = 9x - 3 = 3(3x - 1)$$

$$C = 4x^2 - 9 = (2x)^2 - 3^2 = (2x - 3)(2x + 3)$$

$$D = 1 - (1 - 3x)^2 = 1^2 - (1 - 3x)^2 = (1 - (1 - 3x))(1 + (1 - 3x))$$

$$D = (1 - 1 + 3x)(1 + 1 - 3x) = 3x(2 - 3x)$$

$$E = x^3 - x^2 = x^2(x - 1)$$

$$F = 16x^2 - 8x + 1 = (4x)^2 - 2 \times 4x \times 1 + 1^2 = (4x - 1)^2$$

$$G = 16 - 25x^2 = (4)^2 - (5x)^2 = (4 - 5x)(4 + 5x)$$

$$H = 4x^2 + 4x + 1 = (2x)^2 + 2 \times 2x \times 1 + 1^2 = (2x + 1)^2$$

تمرين 8: عمل التعبيرات التالية: $A = 6x^3 - 2x$ و

$$B = 12x - 4$$

$$E = x^4 - 2x^2 \quad \text{و} \quad D = 4 - (2 - x)^2 \quad \text{و} \quad C = 25x^2 - 16$$

$$H = 25x^2 + 20x + 4 \quad \text{و} \quad G = 1 - 4x^2 \quad F = 9x^2 - 6x + 1$$

أجوبة:

$$A = 6x^3 - 2x = 2x(3x^2 - 1)$$

$$B = 12x - 4 = 4(3x - 1)$$

$$C = 25x^2 - 16 = (5x)^2 - 4^2 = (5x - 4)(5x + 4)$$

$$D = 4 - (2 - x)^2 = 2^2 - (2 - x)^2 = (2 - (2 - x))(2 + (2 - x))$$

$$D = (2 - 2 + x)(2 + 2 - x) = x(4 - x)$$

$$E = x^4 - 2x^2 = x^2(x^2 - 2)$$

$$F = 9x^2 - 6x + 1 = (3x)^2 - 2 \times 3x \times 1 + 1^2 = (3x - 1)^2$$

$$G = 1 - 4x^2 = (1)^2 - (2x)^2 = (1 - 2x)(1 + 2x)$$

$$H = 25x^2 + 20x + 4 = (5x)^2 + 2 \times 5x \times 2 + 2^2 = (5x + 2)^2$$

تمرين 9: أحسب وبسط حيث $a \in \mathbb{R}^*$

$$a^5 \times a^3 \times a^1 \times a^{-2}, (a^3)^5, (-2)^5, \left(-\frac{1}{3}\right)^4, \left(\frac{2}{3}\right)^4$$

$$A = \frac{9^2}{3^2} \times \frac{3^5}{81^3} \quad \text{و} \quad \left(\frac{a}{2}\right)^6, (3a)^2, a^6 \times a^{-3}$$

$$C = 2^3 \times (2^2)^4 \times (2^{-5})^3 \quad \text{و} \quad B = (-5)^2 \times (5^2)^4 \times (5^{-5})^3 \times 5^5$$

$$E = \frac{(-2)^3 \times (4^2)^{-1} \times 8}{1024 \times (-16)^{-4}} \quad D = \frac{3^{-5} \times 4^{-2} \times 9}{12^3} \times \frac{9}{2^2}$$

$$(-2)^5 = -32 \quad \text{و} \quad \left(-\frac{1}{3}\right)^4 = \frac{1^4}{3^4} = \frac{1}{81} \quad \text{و} \quad \left(\frac{2}{3}\right)^4 = \frac{2^4}{3^4} = \frac{16}{81}$$

$$a^5 \times a^3 \times a^1 \times a^{-2} = a^{5+3+1+(-2)} = a^7 \quad \text{و} \quad ((a^2)^5)^3 = a^{2 \times 5 \times 3} = a^{30}$$

$$(3a)^2 = 3^2 \times a^2 = 9a^2 \quad \text{و} \quad a^6 \times a^{-3} = a^{6+(-3)} = a^3$$

$$\left(\frac{a}{2}\right)^6 = \frac{a^6}{2^6} = \frac{a^6}{64}$$

تمرين 5: أنشر: $x \in \mathbb{R}$

$$, -2x(3x - 4), (3x - 1)^2, (\sqrt{2} - 3)^2, (2x + 1)^2$$

$$(3x - 2)(x + 3)$$

$$, (x - 1)^3, \left(\frac{2}{3}x - \frac{3}{5}\right)\left(\frac{2}{3}x + \frac{3}{5}\right), (3\sqrt{3} - 2)(3\sqrt{3} + 2)$$

$$(3x - 2)^3 \quad \text{و} \quad (2x - 1)^3, (2x + 1)^3$$

الأجوبة:

$$(2x + 1)^2 = (2x)^2 + 2 \times 2x \times 1 + 1^2 = 4x^2 + 4x + 1$$

$$(\sqrt{2} - 3)^2 = (\sqrt{2})^2 - 2 \times \sqrt{2} \times 3 + 3^2 = 2 - 6\sqrt{2} + 9 = 11 - 6\sqrt{2}$$

$$(3x - 1)^2 = (3x)^2 - 2 \times 3x \times 1 + 1^2 = 9x^2 - 6x + 1$$

$$-2x(3x - 4) = -6x^2 + 8x$$

$$(3x - 2)(x + 3) = 3x^2 + 9x - 2x - 6 = 3x^2 + 7x - 6$$

$$(3\sqrt{3} - 2)(3\sqrt{3} + 2) = (3\sqrt{3})^2 - 2^2 = 27 - 4 = 23$$

$$\left(\frac{2}{3}x - \frac{3}{5}\right)\left(\frac{2}{3}x + \frac{3}{5}\right) = \left(\frac{2}{3}x\right)^2 - \left(\frac{3}{5}\right)^2 = \frac{4}{9}x^2 - \frac{9}{25}$$

$$(x - 1)^3 = (x)^3 - 3(x)^2 \times 1 + 3 \times x \times (1)^2 - (1)^3 = x^3 - 3x^2 + 3x - 1$$

$$(2x + 1)^3 = (2x)^3 + 3(2x)^2 \times 1 + 3 \times 2x \times (1)^2 + (1)^3 = 8x^3 + 12x^2 + 6x + 1$$

$$(2x - 1)^3 = (2x)^3 - 3(2x)^2 \times 1 + 3 \times 2x \times (1)^2 - (1)^3 = 8x^3 - 12x^2 + 6x - 1$$

$$(3x - 2)^3 = (3x)^3 - 3(3x)^2 \times 2 + 3 \times 3x \times (2)^2 - (2)^3 = 27x^3 - 54x^2 + 36x - 8$$

تمرين 6: أنشر: $x \in \mathbb{R}$

$$-3x(4x - 2), (5x - 2)^2, (\sqrt{3} - 2)^2, (3x + 1)^2$$

$$\left(\frac{1}{2}x - \frac{2}{3}\right)\left(\frac{1}{2}x + \frac{2}{3}\right), (2\sqrt{5} - 3)(2\sqrt{5} + 3), (5x - 2)(2x + 1)$$

$$(2x - 5)^3 \quad \text{و} \quad (3x - 1)^3, (3x + 1)^3, (x - 2)^3$$

$$(3x + 1)^2 = (3x)^2 + 2 \times 3x \times 1 + 1^2 = 9x^2 + 6x + 1$$

$$(\sqrt{3} - 2)^2 = (\sqrt{3})^2 - 2 \times \sqrt{3} \times 2 + 2^2 = 3 - 4\sqrt{3} + 4 = 7 - 4\sqrt{3}$$

$$(3x - 1)^2 = (3x)^2 - 2 \times 3x \times 1 + 1^2 = 9x^2 - 6x + 1$$

$$-3x(4x - 2) = -12x^2 + 6x$$

$$(5x - 2)(2x + 1) = 10x^2 + 5x - 4x - 2 = 10x^2 + x - 2$$

$$(2\sqrt{5} - 3)(2\sqrt{5} + 3) = (2\sqrt{5})^2 - 3^2 = 20 - 9 = 11$$

$$\left(\frac{1}{2}x - \frac{2}{3}\right)\left(\frac{1}{2}x + \frac{2}{3}\right) = \left(\frac{1}{2}x\right)^2 - \left(\frac{2}{3}\right)^2 = \frac{1}{4}x^2 - \frac{4}{9}$$

$$(x - 2)^3 = (x)^3 - 3(x)^2 \times 2 + 3 \times x \times (2)^2 - (2)^3 = x^3 - 6x^2 + 12x - 8$$

$$(3x + 1)^3 = (3x)^3 + 3(3x)^2 \times 1 + 3 \times 3x \times (1)^2 + (1)^3 = 27x^3 + 27x^2 + 9x + 1$$

$$(3x - 1)^3 = (3x)^3 - 3(3x)^2 \times 1 + 3 \times 3x \times (1)^2 - (1)^3 = 27x^3 - 27x^2 + 9x - 1$$

$$(2x - 5)^3 = (2x)^3 - 3(2x)^2 \times 5 + 3 \times 2x \times (5)^2 - (5)^3 = 8x^3 - 60x^2 + 150x - 125$$

تمرين 7: عمل التعبيرات التالية: $A = 3x^2 - 3x$

$$B = 9x - 3$$

$$E = x^3 - x^2 \quad \text{و} \quad D = 1 - (1 - 3x)^2 \quad \text{و} \quad C = 4x^2 - 9$$

$$H = 4x^2 + 4x + 1 \quad \text{و} \quad G = 16 - 25x^2 \quad F = 16x^2 - 8x + 1$$

$$E = \frac{3^{-2} \times (9^2)^3 \times 8}{27 \times 81 \times (-3)^6} = \frac{3^{-2} \times 3^{12} \times 2^3}{3^3 \times 3^4 \times 3^6} = 3^{-2} \times 3^{12} \times 2^3 \times 3^{-3} \times 3^{-4} \times 3^{-6} = 3^0 = 1$$

تمرين 11: (1) أحسب: 11^{-2} , 10^{-3}

(2) أكتب على شكل قوة: 100000 و 0,0001 و 0,002

الجواب: (1) $10^{-3} = \frac{1}{10^3} = 0.001$ و $10^{-2} = \frac{1}{10^2} = 0.01$

(2) $100000 = 10^5$ و $0,0001 = 10^{-4}$ و $0,002 = 2 \times 10^{-3}$

تمرين 12: أحسب وبسط: $A = \frac{(10^6)^4 \times 10^{-2}}{10^4 \times 10^6 \times 10^{-13}}$ و

$$B = \frac{10^{-8} \times 10^9 \times 10^7 \times 10^{-4}}{10^{-2} \times 10^3 \times 10^5}$$

الجواب:

$$A = \frac{(10^6)^4 \times 10^{-2}}{10^4 \times 10^6 \times 10^{-13}} = 10^{24} \times 10^{-2} \times 10^{-4} \times 10^{-6} \times 10^{13} = 10^{-1} = \frac{1}{10} = 0.1$$

$$B = \frac{10^{-8} \times 10^9 \times 10^7 \times 10^{-4}}{10^{-2} \times 10^3 \times 10^5} = 10^{-8} \times 10^9 \times 10^7 \times 10^{-4} \times 10^2 \times 10^{-3} \times 10^{-5}$$

$$B = 10^{-8+9+7-4+2-3-5} = 10^{-2} = \frac{1}{10^2} = \frac{1}{100} = 0.01$$

تمرين 13: أجب بصحيح أو خطأ

• الكتابة العلمية للعدد 149597870 كلم

• هي $1,4959787 \times 10^8$ كلم.

• $3,25 \times 10^4$ هي كتابة علمية

• 15×10^3 هي كتابة علمية

• الكتابة العلمية للعدد -17000000 هي -1.7×10^7

الجواب: الكتابة العلمية تكون على الشكل $x = a \times 10^p$ أو

$$x = -a \times 10^p$$

حيث p ينتمي الى \mathbb{Z} و a عدد عشري بحيث $1 \leq a < 10$ ومنه

• $1,4959787 \times 10^8$ هي كتابة علمية صحيح

• $3,25 \times 10^4$ هي كتابة علمية صحيح

• 15×10^3 هي كتابة علمية خطأ

• -1.7×10^7 هي كتابة علمية صحيح

تمرين 14: حدد الكتابة العلمية للأعداد التالية

300 000 و 45 000 0 و 0,01 و 0,001 و 368 100 و 0,0002 و 25 000 000,000 0

الجواب: $300000 = 3 \times 10^5$ و $450000 = 4.5 \times 10^5$

$0,001 = 10^{-3}$ و $0,01 = 10^{-2}$ و $368100000 = 3.681 \times 10^9$

$0,0002 = 2 \times 10^{-4}$ و $25000000 = 2.5 \times 10^7$

تمرين 15: بسط و أحسب

$$D = \frac{\sqrt{28}}{\sqrt{14}} \quad C = \sqrt{\frac{9}{2}} \quad B = \sqrt{8} \quad A = \sqrt{16}$$

$$E = 3\sqrt{20} + 4\sqrt{45} - 2\sqrt{80} - \sqrt{180}$$

$$G = (\sqrt{5} + \sqrt{2})^2 - (\sqrt{5} - \sqrt{2})^2 \quad F = \frac{\sqrt{3} + \sqrt{5}}{\sqrt{3} - \sqrt{5}} - \frac{\sqrt{3} - \sqrt{5}}{\sqrt{3} + \sqrt{5}} \quad \text{و}$$

$$H = [(\sqrt{2} - \sqrt{3})(\sqrt{2} + \sqrt{3})]^2$$

$$M = (200520052006)^2 - (200520052005 \times 200520052007)$$

عندما تعجز الآلة الحاسبة

الجواب: $A = \sqrt{16} = 4$ و $B = \sqrt{8} = \sqrt{2 \times 4} = \sqrt{2} \times \sqrt{4} = 2\sqrt{2}$

$$A = \frac{9}{3^{-2}} \times \frac{3^5}{81^3} = \frac{3^2}{3^{-2}} \times \frac{3^5}{9^3} = \frac{3^2}{3^{-2}} \times \frac{3^5}{(3^2)^3}$$

$$A = \frac{3^2 \times 3^5}{3^2 \times (3^2)^3} = \frac{3^2 \times 3^5}{3^2 \times 3^6} = 3^2 \times 3^5 \times 3^2 \times 3^{-6} = 3^{2+5+2-6} = 3^3 = 27$$

$$B = (-5)^2 \times (5^2)^4 \times (5^{-5})^3 \times 5^5 = 5^2 \times (5^2)^4 \times (5^{-5})^3 \times 5^5$$

$$B = 5^2 \times 5^{2 \times 4} \times 5^{-5 \times 3} \times 5^5 = 5^{2+8-15+5} = 5^0 = 1$$

$$C = 2^3 \times (2^2)^4 \times (2^{-5})^3 = 2^3 \times 2^8 \times 2^{-15} = 2^{3+8-15} = 2^{-4} = \frac{1}{2^4} = \frac{1}{16}$$

$$D = \frac{3^{-5} \times 4^{-2}}{12^3} \times \frac{9}{2^2} = \frac{3^{-5} \times (2^2)^{-2}}{(3 \times 2^2)^3} \times \frac{3^2}{2^2} = \frac{3^{-5} \times (2)^{-4} \times 3^2}{(3)^3 \times 2^6 \times 2^2}$$

$$D = \frac{3^{-5} \times (2)^{-4} \times 3^2}{(3)^3 \times 2^6 \times 2^2} = 3^{-5} \times 2^{-4} \times 3^2 \times (3)^{-3} \times 2^{-6} \times 2^{-2} = 3^{-5-3+2} \times 2^{-4-6-2}$$

$$D = 3^{-6} \times 2^{-12}$$

$$E = \frac{(-2)^3 \times (4^2)^{-1} \times 8}{1024 \times (-16)^{-4}} = \frac{-2^3 \times 4^{2 \times (-1)} \times 2^3}{1024 \times (-2^3)^{-4}} = \frac{-2^3 \times (2^2)^{-2} \times 2^3}{2^{10} \times (-2^3)^{-4}}$$

$$E = -2^3 \times (2^2)^{-2} \times 2^3 \times 2^{-10} \times (-2)^{3 \times 4} = -2^{3-4+3-10+12} = -2^{-4} = -16$$

تمرين 10: أحسب وبسط حيث $a \in \mathbb{R}^*$

$$a^7 \times a^{-3} \times a^5 \times a^{-8}, ((a)^{-2})^3, (-5)^3, \left(-\frac{3}{2}\right)^4, \left(-\frac{3}{5}\right)^3$$

$$A = \frac{4}{2^{-3}} \times \frac{2^5}{16^3} \quad \text{و} \quad \left(\frac{a}{3}\right)^5, (5a)^3, a^7 \times a^{-9}$$

$$C = 2^7 \times (2^5)^2 \times (2^{-5})^3 \quad \text{و} \quad B = (-3)^2 \times (3^2)^4 \times (3^{-5})^3 \times 3^4$$

$$E = \frac{3^2 \times (9^2)^3 \times 8}{27 \times 81 \times (-3)^6} \quad D = \frac{5 \times 3^{-2}}{15^2} \times \frac{81}{5^{-4}}$$

الجواب:

$$\left(-\frac{3}{2}\right)^4 = \frac{3^4}{2^4} = \frac{81}{16} \quad \text{و} \quad \left(-\frac{3}{5}\right)^3 = -\frac{3^3}{5^3} = -\frac{27}{125}$$

$$(-5)^3 = -5^3 = -125$$

$$a^7 \times a^{-3} \times a^5 \times a^{-8} = a^{7-3+5+(-8)} = a^1 = a \quad \text{و} \quad ((a)^{-2})^3 = a^{-2 \times 3} = a^{-6}$$

$$a^7 \times a^{-9} = a^{7+(-9)} = a^{-2} = \frac{1}{a^2}$$

$$(5a)^3 = 5^3 \times a^3 = 125a^3$$

$$\left(\frac{a}{3}\right)^5 = \frac{a^5}{3^5} = \frac{a^5}{243}$$

$$A = \frac{4}{2^{-3}} \times \frac{2^5}{16^3} = \frac{2^2 \times 2^5}{2^{-3} \times 2^{12}} = 2^2 \times 2^5 \times 2^3 \times 2^{-12} = 2^{2+5+3-12} = 2^{-2} = \frac{1}{2^2} = \frac{1}{4}$$

$$B = (-3)^2 \times (3^2)^4 \times (3^{-5})^3 \times 3^4 = 3^2 \times (3^2)^4 \times (3^{-5})^3 \times 3^4$$

$$B = 3^2 \times 3^{2 \times 4} \times 3^{-5 \times 3} \times 3^4 = 3^{2+8-15+4} = 3^{-1} = \frac{1}{3}$$

$$C = 2^7 \times (2^5)^2 \times (2^{-5})^3 = 2^7 \times 2^{10} \times 2^{-15} = 2^{7+10-15} = 2^2 = 4$$

$$D = \frac{5 \times 3^{-2}}{15^2} \times \frac{81}{5^4} = \frac{5^1 \times 3^{-2} \times (3)^4}{(3 \times 5)^2 \times 5^4} = 5^1 \times 3^{-2} \times 3^4 \times 3^{-2} \times 5^{-2} \times 5^4 = 5^3 = 125$$

$$E = 2\sqrt{2}$$

$$F = \frac{5\sqrt{7}}{\sqrt{2}-\sqrt{7}} + \frac{5\sqrt{2}}{\sqrt{2}+\sqrt{7}} = \frac{(5\sqrt{7})(\sqrt{2}+\sqrt{7}) + 5\sqrt{2}(\sqrt{2}-\sqrt{7})}{(\sqrt{2}+\sqrt{7})(\sqrt{2}-\sqrt{7})}$$

$$F = \frac{5\sqrt{7}\sqrt{2} + 5\sqrt{7}\sqrt{7} + 5\sqrt{2}\sqrt{2} - 5\sqrt{2}\sqrt{7}}{(\sqrt{2})^2 - (\sqrt{7})^2} = \frac{35 + 10}{(\sqrt{2})^2 - (\sqrt{7})^2} = \frac{45}{-5} = -9$$

$$G = (\sqrt{3} + \sqrt{11})^2 - (\sqrt{3} - \sqrt{11})^2 = (\sqrt{3})^2 + 2\sqrt{3}\sqrt{11} + (\sqrt{11})^2 - ((\sqrt{3})^2 - 2\sqrt{3}\sqrt{11} + (\sqrt{11})^2)$$

$$G = 3 + 2\sqrt{33} + 11 - (3 - 2\sqrt{33} + 11) = 3 + 2\sqrt{33} + 11 - 3 + 2\sqrt{33} - 11 = 4\sqrt{33}$$

$$H = [(\sqrt{2} - \sqrt{7})(\sqrt{2} + \sqrt{7})]^2 = [(\sqrt{2})^2 - (\sqrt{7})^2]^2$$

$$H = [2 - 7]^2 = (-5)^2 = 25$$

$$M = ((4\sqrt{3} - 7)(4\sqrt{3} + 7))^{2015} = ((4\sqrt{3})^2 - (7)^2)^{2015} = (48 - 49)^{2015} = (-1)^{2015} = -1$$

$$N = (\sqrt{75} - \sqrt{98}) \times (5\sqrt{3} + 7\sqrt{2}) = (\sqrt{25 \times 3} - \sqrt{49 \times 2}) \times (5\sqrt{3} + 7\sqrt{2})$$

$$N = (5\sqrt{3} - 7\sqrt{2}) \times (5\sqrt{3} + 7\sqrt{2}) = (5\sqrt{3})^2 - (7\sqrt{2})^2$$

$$N = 75 - 98 = 75 - 98 = -23$$

$$P = (5x + 2)^3 = (5x)^3 + 3(5x)^2 \times 2 + 3 \times 5x \times (2)^2 + (2)^3$$

$$P = 125x^3 + 150x^2 + 60x + 8$$

$$Q = (\sqrt{3} + 1)^3 = (\sqrt{3})^3 + 3(\sqrt{3})^2 \times 1 + 3 \times \sqrt{3} \times (1)^2 + (1)^3$$

$$Q = 3\sqrt{3} + 9 + 3\sqrt{3} + 1 = 6\sqrt{3} + 10$$

$$G = (2015200052004)^2 - (2015200052002 \times 2015200052006)$$

نلاحظ أن الأعداد الثلاثة تختلف فقط في رقم وحداتها

لتبسيط الحساب نضع: $x = 2015200052004$

إذن: $x + 2 = 2015200052006$ و $x - 2 = 2015200052002$

و منه: $G = x^2 - (x - 2)(x + 2)$

$$G = x^2 - (x^2 - 4) = x^2 - x^2 + 4 = 4$$

تمرين 17: حدد العدد الحقيقي x إذا علمت أن الأعداد:

$x + 1$ و $x + 3$ و x و 2 تكون في هذا الترتيب تناسبا

الجواب: $x + 1$ و $x + 3$ و x و 2 تكون في هذا الترتيب تناسبا

$$2(x + 1) = 3x \text{ يعني } \frac{x + 1}{3} = \frac{x}{2} \text{ يعني}$$

$$x = 2 \text{ يعني } -x = -2 \text{ يعني } 2x + 2 = 3x$$

« c'est en forgeant que l'on devient forgeron » dit un proverbe.
c'est en s'entraînant

régulièrement aux calculs et
exercices que l'on devient un



$$C = \sqrt{\frac{9}{2}} = \frac{\sqrt{9}}{\sqrt{2}} = \frac{3}{\sqrt{2}} = \frac{3\sqrt{2}}{\sqrt{2} \times \sqrt{2}} = \frac{3\sqrt{2}}{(\sqrt{2})^2} = \frac{3\sqrt{2}}{2}$$

$$D = \frac{\sqrt{28}}{\sqrt{14}} = \sqrt{\frac{28}{14}} = \sqrt{2}$$

$$E = 3\sqrt{20} + 4\sqrt{45} - 2\sqrt{80} - \sqrt{180} = 3\sqrt{4 \times 5} + 4\sqrt{9 \times 5} - 2\sqrt{16 \times 5} - \sqrt{36 \times 5}$$

$$E = 3 \times 2\sqrt{5} + 4 \times 3\sqrt{5} - 2 \times 4\sqrt{5} - 6\sqrt{5} = 6\sqrt{5} + 12\sqrt{5} - 8\sqrt{5} - 6\sqrt{5} = (6 + 12 - 8 - 6)\sqrt{5}$$

$$E = 4\sqrt{5}$$

$$F = \frac{\sqrt{3} + \sqrt{5}}{\sqrt{3} - \sqrt{5}} - \frac{\sqrt{3} - \sqrt{5}}{\sqrt{3} + \sqrt{5}} = \frac{(\sqrt{3} + \sqrt{5})(\sqrt{3} + \sqrt{5}) - (\sqrt{3} - \sqrt{5})(\sqrt{3} - \sqrt{5})}{(\sqrt{3} + \sqrt{5})(\sqrt{3} - \sqrt{5})}$$

$$F = \frac{(\sqrt{3} + \sqrt{5})^2 - (\sqrt{3} - \sqrt{5})^2}{(\sqrt{3})^2 - (\sqrt{5})^2} = \frac{(\sqrt{3})^2 + 2\sqrt{3}\sqrt{5} + (\sqrt{5})^2 - ((\sqrt{3})^2 - 2\sqrt{3}\sqrt{5} + (\sqrt{5})^2)}{(\sqrt{3})^2 - (\sqrt{5})^2}$$

$$F = \frac{3 + 2\sqrt{15} + 5 - (3 - 2\sqrt{15} + 5)}{(\sqrt{3})^2 - (\sqrt{5})^2} = \frac{3 + 2\sqrt{15} + 5 - 3 + 2\sqrt{15} - 5}{(\sqrt{3})^2 - (\sqrt{5})^2} = \frac{4\sqrt{15}}{-2} = -2\sqrt{15}$$

$$G = (\sqrt{5} + \sqrt{2})^2 - (\sqrt{5} - \sqrt{2})^2 = (\sqrt{5})^2 + 2\sqrt{5}\sqrt{2} + (\sqrt{2})^2 - ((\sqrt{5})^2 - 2\sqrt{5}\sqrt{2} + (\sqrt{2})^2)$$

$$G = 5 + 2\sqrt{10} + 2 - (5 - 2\sqrt{10} + 2) = 5 + 2\sqrt{10} + 2 - 5 + 2\sqrt{10} - 2 = 4\sqrt{10}$$

$$H = [(\sqrt{2} - \sqrt{3})(\sqrt{2} + \sqrt{3})]^2 = [(\sqrt{2})^2 - (\sqrt{3})^2]^2 = (2 - 3)^2 = (-1)^2 = 1$$

$$M = (200520052006)^2 - (200520052005 \times 200520052007)$$

حساب: نلاحظ أن الأعداد الثلاثة تختلف فقط في رقم وحداتها لتبسيط الحساب

نضع: $x = 200520052006$

إذن: $200520052007 = x + 1$ و $200520052005 = x - 1$

و منه: $M = x^2 - (x - 1)(x + 1)$

$$M = x^2 - (x^2 - 1) = x^2 - x^2 + 1 = 1$$

تمرين 16: بسط و أحسب

$$D = \frac{\sqrt{60}}{\sqrt{15}} \quad C = \sqrt{\frac{16}{3}} \quad B = \sqrt{50} \quad A = \sqrt{121}$$

$$E = 6\sqrt{8} + \sqrt{32} - 2\sqrt{98}$$

$$G = (\sqrt{7} + \sqrt{2})^2 - (\sqrt{7} - \sqrt{2})^2 \quad F = \frac{5\sqrt{7}}{\sqrt{2} - \sqrt{7}} + \frac{5\sqrt{2}}{\sqrt{2} + \sqrt{7}}$$

$$H = [(\sqrt{2} - \sqrt{7})(\sqrt{2} + \sqrt{7})]^2$$

$$M = (4\sqrt{3} - 7)^{2015} \times (4\sqrt{3} + 7)^{2015} \quad G = (\sqrt{3} + \sqrt{11})^2 - (\sqrt{3} - \sqrt{11})^2$$

$$Q = (\sqrt{3} + 1)^3 \quad P = (5x + 2)^3 \quad N = (\sqrt{75} - \sqrt{98}) \times (5\sqrt{3} + 7\sqrt{2})$$

$$G = (2015200052004)^2 - (2015200052002 \times 2015200052006)$$

الجواب:

$$B = \sqrt{50} = \sqrt{2 \times 25} = \sqrt{2} \times \sqrt{25} = 5\sqrt{2} \quad \text{و} \quad A = \sqrt{121} = 11$$

$$C = \sqrt{\frac{16}{3}} = \frac{\sqrt{16}}{\sqrt{3}} = \frac{4}{\sqrt{3}} = \frac{4\sqrt{3}}{\sqrt{3} \times \sqrt{3}} = \frac{4\sqrt{3}}{(\sqrt{3})^2} = \frac{4\sqrt{3}}{3} = 2\sqrt{2}$$

$$D = \frac{\sqrt{60}}{\sqrt{15}} = \sqrt{\frac{60}{15}} = \sqrt{4} = 2$$

$$E = 6\sqrt{8} + \sqrt{32} - 2\sqrt{98} = 6\sqrt{4 \times 2} + \sqrt{16 \times 2} - 2\sqrt{49 \times 2}$$

$$E = 12\sqrt{2} + 4\sqrt{2} - 14\sqrt{2}$$